The role of discreteness in the Black Hole Information Paradox

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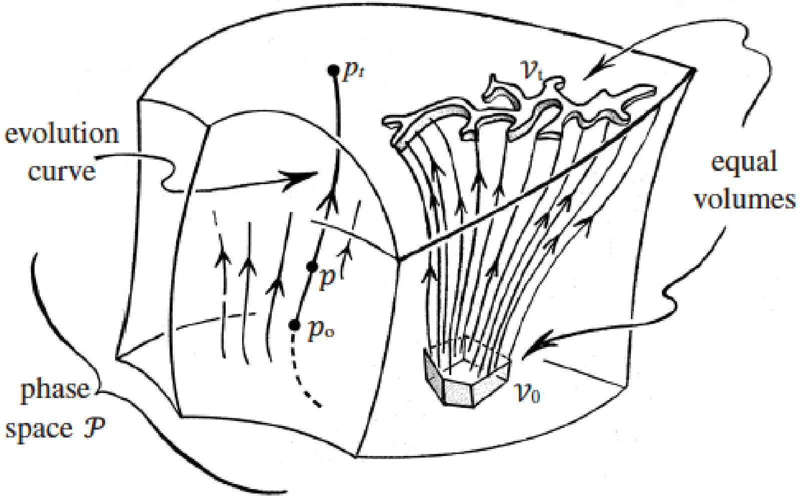
Entropy, Irreversibility and Coarse Graining



Penrose (2010)

Special initial conditions

Complexity

Coarse Graining

Black Holes and Thermodynamics

- **Zeroth Law:** κ is constant over the horizon of a stationary black hole
- **First Law:** First order variations to equilibrium states

$$dM = \underbrace{\frac{1}{8\pi M}}_{Heat!} dA + \underbrace{\Omega_h dJ + \Phi dQ}_{\text{work terms}} \tag{1}$$

- **Second Law:** $\delta A \geq 0$ ($\delta S_{bh} + \delta S_{matter} \geq 0$) in any process
- **Third Law:** $\kappa = 0$ is impossible to attain by any physical process

Black Holes Entropy - An approach from LQG

$$S_{bh} \propto A$$

Rovelli, Ashtekar, Krasnov

Baez, Corichi, Barbero, etc

Key Insight: Each classical black hole geometry (macrostate) is compatible with a lot of quantum geometry states in the fundamental theory (microstates)

Black Holes Entropy - An approach from LQG

$$S_{bh} \propto A$$

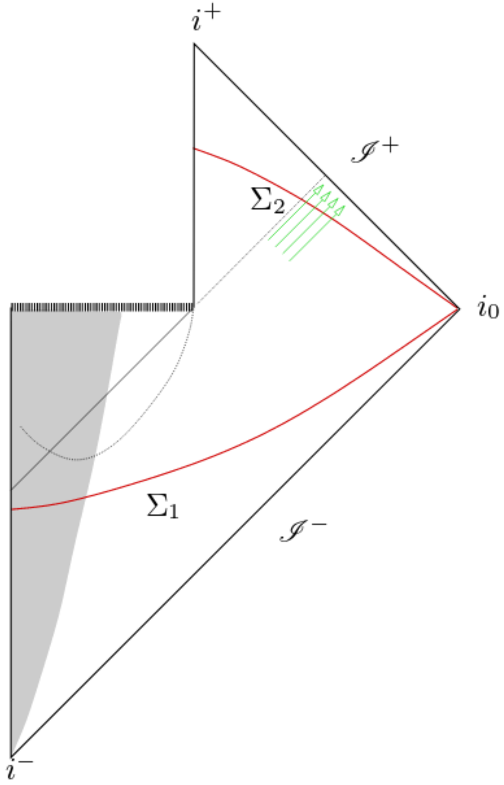
Rovelli, Ashtekar, Krasnov

Baez, Corichi, Barbero, etc

Assumption: This is true in general. Each macroscopic smooth geometry is **highly degenerate** in the fundamental theory. Smoothness arises from coarse graning of these quantum geometries.

Key for solving Hawking's information paradox

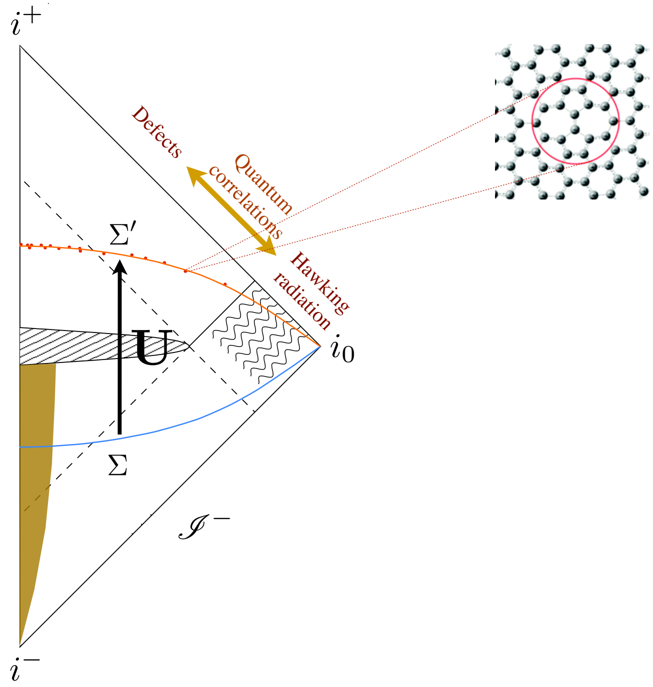
Black Hole Evaporation - Information Paradox



Σ_2 is **not** a Cauchy surface → non unitary evolution

BH Evaporation - The role of discreteness

- Initially the black hole is arbitrarily large → **large entropy in Hawking radiation** needs **large number of d.o.f** for purification
- We only have m_{Pl} to do that!



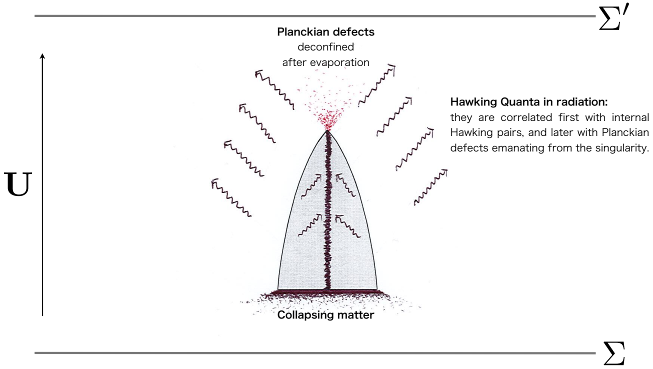
Microstructure at Planck scale (Planckian *defects*) provides an enourmous reservoir **no energy cost**

If just Hawking radiation is taken into account there is an **entropy jump**

Alejandro Perez, CQG, 32 (2015)

BH Evaporation - The role of discreteness

Planckian microstructure is hidden for **low-energy coarse-grained** observers


Σ'
Planckian defects
deconfined
after evaporation
Hawking Quanta in radiation:
they are correlated first with internal Hawking pairs, and later with Planckian defects emanating from the singularity.
U
Collapsing matter
Σ

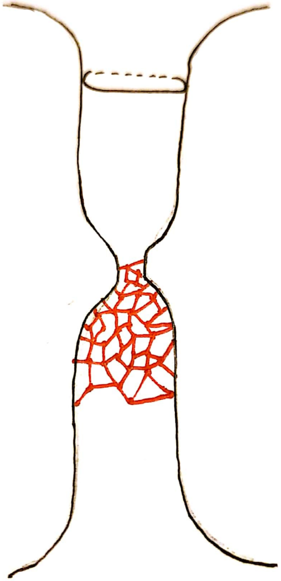
Evolution can be unitary in the fundamental theory if d.o.f. responsible for BH entropy in LQG are taken into account.

These d.o.f. are not describable in terms of QFT on CS

Alejandro Perez, CQG, 32 (2015)

A toy model realized in Quantum Cosmology

These expectations can be made precise in **Loop Quantum Cosmology** LA and A. Perez (2019)



Special initial conditions: **Planckian defects are uncorrelated initially**

During the high curvature phase **correlations with Planckian d.o.f. are established**

These d.o.f. **do not carry any energy**

For **coarse-grained observers** evolution seems to be **not unitary**

Loop (Polymer) Quantization

$$\mathcal{H}_{Poly} = \bigoplus_{\gamma} \mathcal{H}_{\gamma} = \bigoplus_{\epsilon \in [0,4k)} \mathcal{H}_{\Gamma_\epsilon}$$

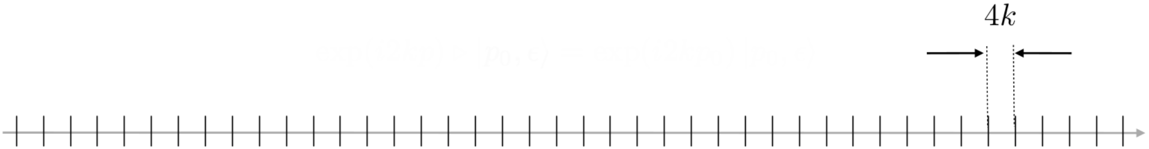
$$\Gamma_\epsilon = \{4kn + \epsilon, n \in \mathbb{Z}, \epsilon \in [0,4k)\}$$

$\hat{p}$ is not defined on $\mathcal{H}_{Poly}$ → only finite translations $\widehat{e^{ikp}}$ are defined

$$\widehat{e^{2ikp}} \triangleright \Psi_\gamma(x) = \Psi_\gamma(x - 4k)$$

Eigenvectors of the Shift operators → $|p_0, \Gamma^k_\epsilon\rangle$

$$\langle x | p_0, \Gamma^k_\epsilon \rangle = \sum_{n \in \mathbb{Z}} e^{-i\frac{p_0}{2}x} \delta_{x,4kn+\epsilon}, \qquad \widehat{e^{2ikp}} \triangleright |p_0, \Gamma^k_\epsilon\rangle = e^{2ikp_0} |p_0, \Gamma^k_\epsilon\rangle \qquad \epsilon \in [0,4k)$$



$$\exp(i2kp) \triangleright |p_0, \epsilon\rangle = \exp(i2kp_0) |p_0, \epsilon\rangle$$

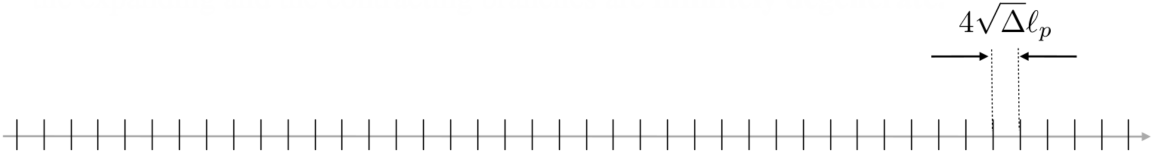
Loop (Polymer) Quantization

The regularized Hamiltonian becomes a **free particle-like system** in polymer quantum mechanics:

$$H = \frac{3}{\gamma^2} p^2 \xrightarrow{\text{UV input } \Delta} \hat{\Lambda}_\Delta \equiv \frac{3}{\gamma^2 \Delta \ell_p^2} \widehat{\sin^2\left(\Delta^{\frac{1}{2}} \ell_p \, p\right)} \tag{2}$$

The eigenstates of the cosmological constant are given by $|p_0, \Gamma_\epsilon^k\rangle$ with $k = \sqrt{\Delta}\ell_p \rightarrow |p_0, \Gamma_\epsilon^{\sqrt{\Delta}\ell_p}\rangle \equiv |p_0, \Gamma_\epsilon\rangle$

$$\hat{\Lambda}_\Delta \, |p_0, \Gamma_\epsilon\rangle = \Lambda_\Delta(p_0) \, |p_0, \Gamma_\epsilon\rangle \;\rightarrow\; \Lambda_\Delta(p_0) = \frac{3}{\gamma^2 \Delta \ell_p^2} \sin^2\left(\Delta^{\frac{1}{2}} \ell_p \, p_0\right) \tag{3}$$



The eigenstates of the cosmological constant are **infinitely degenerated** (they do not depend on ϵ)

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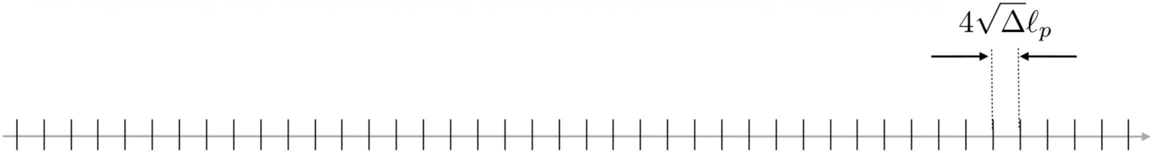

In the WdW quantization the eigenstates of the cosmological constant are **two-fold degenerate** → Universe is either expanding or contracting

Loop (Polymer) Quantization

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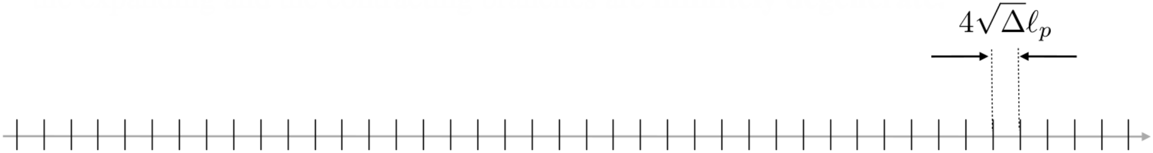


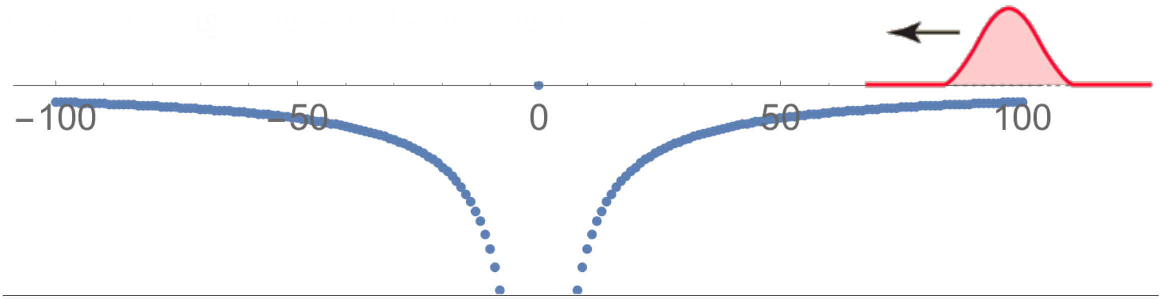
Coarse Graining!

Matter Coupling: Scalar field

$$\text{Matter Hamiltonian} \to H_\phi = \frac{p_\phi^2}{8\pi^2\gamma^2\ell_p^4 x^2} \xrightarrow{\text{Regularization}} \hat{H}_\phi$$

$$\left(\hat{\Lambda}_0 - \frac{8\pi G \ell_p^2}{V_0}\hat{H}_\phi\right)|\psi\rangle = \Lambda\,|\psi\rangle$$

If $|\psi\rangle$ is an eigenstate of the momentum operator $p_\phi \to$ **Scattering problem**

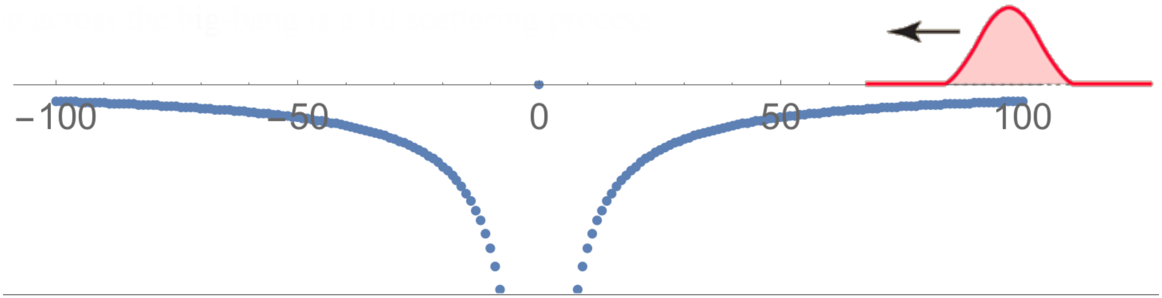


Matter Coupling: Scalar field

Key insight: Different lattices "see" different potentials

For simplicity, we will consider the superposition of only two lattices

$$\Gamma_1 := \Gamma^{\sqrt{\Delta}\ell_p}_{\epsilon=0} \text{ and } \Gamma_2 := \Gamma^{\sqrt{\Delta}\ell_p}_{\epsilon=2\sqrt{\Delta}\ell_p}$$



Scalar Field - Entropy

$$\rho(t) = |\Psi(t)\rangle \langle\Psi(t)|$$

$$\rho_{\text{in}} = \rho(t \to -\infty) \xrightarrow{\text{Unitary Evolution}} \rho_{\text{out}} = \rho(t \to \infty)$$

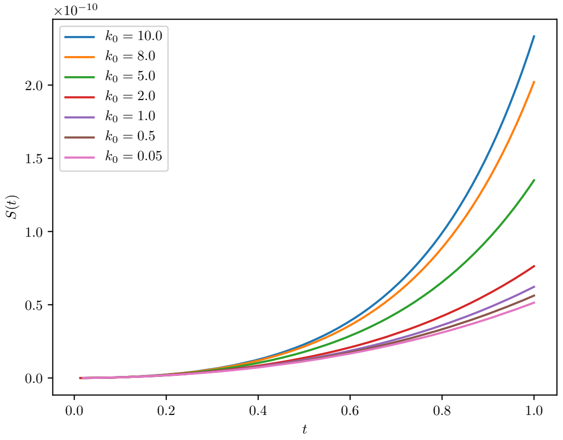
Relevant physical info for coarse-grained observers is encoded in the **reduced density matrix** → $\langle p|\rho^R|p'\rangle = \sum_{i=1}^{2} \langle p, i|\rho|p', i\rangle$

$$\rho^R_{\text{in}} \xrightarrow{\textbf{Non Unitary Evolution}} \rho^R_{\text{out}}$$

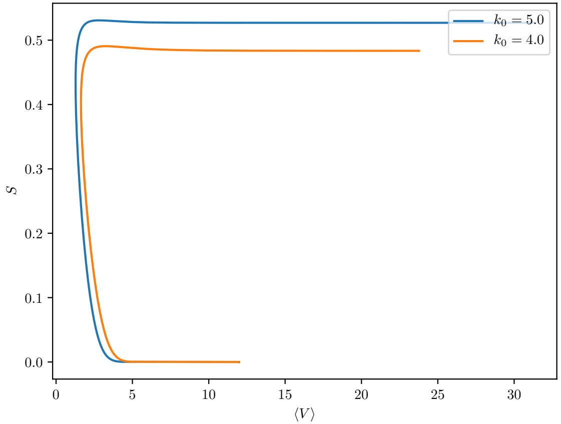
Mixed!

Scalar field - Entropy

Away from the bounce



Through the bounce



The **entropy jumps** at the big-bang no matter how small scalar curvature the initial state has

For low energy coarse-grained observers (insensitive to UV details) pure states evolve into mixed states

These d.o.f. do not carry any energy

Conclusions

Information is not lost but degraded in BH evaporation

- Special initial conditions, complexity and coarse-graibed observers will generically lead to non-unitary evolution.
- When degrees of freedom associated with the discrete quantum nature of spacetime are disregarded, pure states can evolve into mixed states in a low-energy description.
- This idea can be realized in a concrete, fully quantum-gravitational context
- **The consideration of the discrete nature of spacetime at Planck scale is crucial to understand the fate of information in quantum gravity**

Thank you!